Ordinary Differential Equations

Subject -Differential Equations

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Differential Equations:

- A differential equation is an equation which involves derivatives or differential of one or more dependent variable w.r.t one or more independent variables

E.g.: $\boldsymbol{\frac{d^2x}{dt^2} + n^2x = 0}$,

$$x\frac{dy}{dx} + y = x^2y \qquad \frac{d^2y}{dx^2} - y = \cos 4x \qquad x\frac{dz}{dx} + y\frac{dz}{dy} + 2z = 0$$

$$dy = \cos x dx$$

are all differential equations.

Classifications of Differential equations:

Differential equations are classified into two categories "ordinary and partial" depending on the number of independent variables appearing in the equation.

- Ordinary Differential Equations
- Partial Differential Equations

Ordinary Differential Equations

An ordinary differential equation is a differential equation which involve single independent variable and differential co-efficients w.r.t it.

Order of a differential equation:

The order of a differential equation is the order of the highest derivative, appears in given equation.

Degree of a differential equation:

The degree of a differential equation is the power of the highest derivative, which occurs in given equation.

Linear Differential Equation:

A linear differential equation is that in which the dependent variable and its differential coefficient occur in the first degree and are not multiplied together. The standard form of a linear differential equation of the first order is

$$\frac{dy}{dx} + Py = Q$$

where P and Q are the functions of x only.

i. dependent variable y and all its derivatives are of degree one.
ii. No product terms of y and dy/dx or any of it derivatives are present.
iii. No transcendental functions of y and dy/dx or its derivatives occur.

Otherwise, known as Non-Linear differential equations.

Examples

1. $8y'''+2y'+\cos y=e^{x}$
Order-3
Degree-1
Non-Linear

2. $\frac{dy}{dx}+P(x)y=y^{n}Q(x)$
Order-1
Degree-1
Non-linear when y>1

3. $y''-2y=$ Sinx
Order-2
Degree-1
linear

4. $xy'+2y=x^{2}-x+1$

Order- 1
Degree-1
linear

5. $y^{2}dx+(3xy-1)dy=0$

Order-1
Degree-1
Nonlinear in y, linear in x

Linear differential Equations are classified into two types

- Leibnitz’s Linear Equation

- Bernoulli's Equation

Linear differential equations:

The standard form of a linear equation of the first order, commonly known as Leibnitz's linear equation, is

$$\frac{dy}{dx} + P(x)y = Q(x),$$

where P, Q are functions of x only or may be constant.

Solution is $y(I.F.) = \int Q(I.F.)dx + c$

where, I.F.= $e^{\int P dx}$

Example 1:

Solve the differential equation $x\log x\frac{dy}{dx} + y = 2\log x$

Solution:Given equation is $x\log x\frac{dy}{dx} + y = 2\log x$

$$\frac{dy}{dx} + \left(\frac{1}{x\log x}\right)y = \frac{2}{x}$$

Comparing it with $\frac{dy}{dx} + Py = Q$, we have $P =,$ $\frac{1}{x\log x}$ $Q =.$ $\frac{2}{x}$

$$\text{I.F.} = e^{\int Pdx} = e^{\int \frac{1}{x\log x}}$$

$$= \log x$$

Thus, the solution of equation (i) is $y(I.F.) = \int Q(I..F.)dx + c$

$y\ \log x = \int \frac{2}{x} \log x\ dx + c$

$y \log x = (log x)^2 + c$

Which is required solution.

Bernoulli's Equation:

The equation $\frac{dy}{dx} + Py = Qy^n$

where P, Q are functions of x only or constants, reducible to the Leibnitz's linear equation and is usually called the Bernoulli's equation.

If n=0,1 then Bernoulli's equation becomes Leibnitz's linear equation.

Reduction of Bernaulli's equation $\frac{dy}{dx} + Py = Qy^n$ into linear where P *and* Q are the functions of x only.

Reduction

- Divide by y^n ➔ $y^{-n}\frac{dy}{dx} + y^{1-n}P(x) = Q(x)$
- We're going to use u-substitution, so let $u = y^{1-n}$ ➔ $\frac{du}{dx} = (1-n)y^{-n}\frac{dy}{dx}$
- So $\frac{\frac{du}{dx}}{(1-n)} = y^{-n}\frac{dy}{dx}$
- Substitute back into original equation ➔ $\frac{\frac{du}{dx}}{(1-n)} + u(P(x)) = Q(x)$
- Multiply by $(1 - n)$ ➔ $\frac{du}{dx} + (1-n)P(x)u = (1-n)Q(x)$
- The differential equation is now linear (ugly, but linear)!

Example 3:

Solve $x\frac{dy}{dx}+y=x^3y^6$

Solution: The given equation is $x\frac{dy}{dx}+y=x^3y^6$

Dividing throughout by xy^6, we get $y^{-6}\frac{dy}{dx}+\frac{y^{-5}}{x}=x^2$.

Putting $y^{-5}=z$, so that

$$-5y^{-6}\frac{dy}{dx}=\frac{dz}{dx}$$

$$\frac{-1}{5}\frac{dz}{dx}+\frac{z}{x}=x^2$$

$$\Rightarrow \frac{dz}{dx}-\frac{5}{x}z=-5x^2,$$

which is Leibnitz's linear equation in z.

$$\text{I.F.} = e^{-\int\left(\frac{5}{x}\right)dx}$$

$$= e^{-5\,log\,x}$$

$$= e^{log\,x^{-5}} = x^{-5}$$

$$z(I.F.) = \int(-5x^2)(I.F)dx + c$$

$$\Rightarrow zx^{-5} = \int(-5x^2)x^{-5}dx + c$$

$$\Rightarrow y^{-5}x^{-5} = -5.\frac{x^{-2}}{-2} + c$$

Thanks